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Laminar heat transfer in square duct flow of aqueous CMC solutions

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Abstract—Experimental studies on the laminar heat transfer of 4000 wppm and 8000 wppm aqueous carboxymethylcellulose(CMC) solutions in a top-wall heated square duct have been carried out. The square duct is 240 cm long and its hydraulic diameter is 0.4 cm. Results show that the aqueous CMC solutions are good candidates for laminar heat transfer enhancement in a non-circular duct. Under the examined conditions, the enhanced local heat transfer along the whole duct wall can be characterized by a two-zone model with a Newtonian heat transfer section followed by a viscoelastic heat transfer section. The fully developed heat transfer of the aqueous CMC solutions can be well determined by linear correlations with the Kozicki generalized Reynolds number.

1. INTRODUCTION

Although the forced convection heat transfer of polymer solutions, which are usually non-Newtonian fluids, is a very important topic for engineering applications and academic studies, it has been in general not well understood because of the complex nature and limited research. One of the most fascinating phenomena about the problem is that they can give considerable heat transfer enhancement without noticeable friction drag increase in laminar flows through non-circular ducts.

Most of the research work on non-Newtonian fluid heat transfer in laminar flows through non-circular ducts has been conducted with rectangular or square ducts. Reviews on the current knowledge of laminar heat transfer of non-Newtonian fluids in rectangular or square ducts were provided by Lawal and Majumdar [1], Hartnett and Kostic [2], Hartnett [3] and Kostic [4]. It seems that most of the recent research works on the problem were numerical, for example, refs. [5–10]. Since a reasonable model which can capture the physics and predict the heat transfer has not been developed, more experimental work is needed to reveal the heat transfer law of the non-Newtonian fluids.

The non-Newtonian fluids which have been used in the previous experimental studies on the laminar flow heat transfer in rectangular or square ducts, are mainly aqueous solutions of polyacrylamide (Separan), polyacrylic acid (Carbopol), polyethylene oxide (Polyox) and hydroxyethylcellulose (Natrosol). The frequency of use of these polymers in the existing related references is shown in Fig. 1. Typical experiments include that by Kostic [11] and Xie [12] with a 2:1 rectangular duct, by Rao [13] with a 5:1 rectangular duct, and by Mena *et al.* [14] and Lawal [15] with square ducts. The hydraulic diameter D_h range of the rectangular or square ducts used in all of the previous experimental studies is 1.0–3.75 cm.

There is another kind of important polymer, the carboxymethylcellulose (CMC). Although they have not been employed in the study of the laminar heat transfer in non-circular ducts, the aqueous CMC solu-

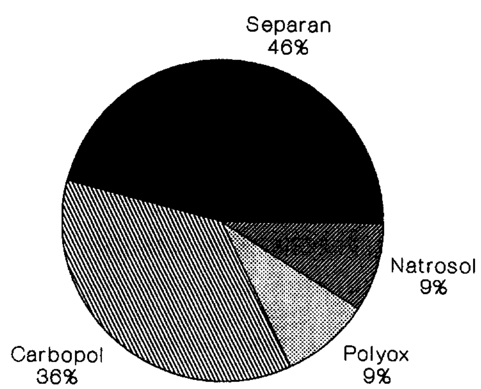

Fig. 1. The polymer-use frequency in the existing experimental studies on the laminar heat transfer of non-Newtonian fluids in rectangular or square ducts.

NOMENCLATURE

C concentration of the polymer solution [wppm]
C_p specific heat of the fluid [J kg^{-1} K^{-1}]
D_h hydraulic diameter of the duct [m]
E relative change of local heat transfer between two polymer solutions, defined in equation (6); also the relative heat transfer increase of non-Newtonian fluid over Newtonian fluid [%]
f fanning friction factor $= \tau_w/(\rho U^2/2)$
Gz Graetz number $= \dot{m}C_p/kx$
h_x local convective heat transfer coefficient $= q/(T_w - T_b)$ [W m^{-2} K^{-1}]
$H1(1)$ thermal boundary condition representing constant wall heat flux axially and constant temperature peripherally on one wall of the square duct
$H1(4)$ thermal boundary condition representing constant wall heat flux axially and constant temperature peripherally on all four walls of the square duct
k thermal conductivity [W m^{-1} K^{-1}]
K consistency index in power-law model [N s^n m^{-2}]
L length of the duct [m]
$\dot{m}$ mass flow rate [kg s^{-1}]
n power-law index in power-law model
Nu_{fd} fully established Nusselt number
Nu_x local Nusselt number $= h_xD_h/k$
q heat flux per unit heating area [W m^{-2}]
Re^* Kozicki generalized Reynolds number, defined in equation (3)
Re^*_{min} minimum Reynolds number Re^* required for polymer solution to start enhancing heat transfer
T_b local bulk temperature of the fluid [°C]
T_w local wall temperature of the duct [°C]
U mean velocity in axial direction [m s^{-1}]
wppm weight parts per million
x axial rectilinear coordinate, or axial location from the duct entrance [m].

Greek symbols
$\dot{\gamma}$ shear rate [s^{-1}]
η fluid apparent viscosity [N s^{-1} m^{-2}]
ρ density of fluid [kg m^{-3}]
τ_w shear stress at wall [N m^{-2}].

tions have been widely used to investigate the drag reduction in turbulent flows [2, 4, 16]. Also, aqueous CMC solutions have been used by Wheeler and Wissler [17] to explore the secondary flow resulting from the viscoelastic behavior of the fluids in a square duct (without heating). It should be mentioned here that many investigators have suggested that the laminar heat transfer enhancement in non-circular ducts with non-Newtonian fluids is due to the secondary flow generated by the elasticity of the fluids.

The present study is to investigate the laminar flow heat transfer of two aqueous CMC solutions in a relatively small-scale, top-heated square duct within low Reynolds number range. The square duct is long enough to obtain both the thermally developing and the thermally developed heat transfer results. Emphasis is focused on the basic patterns of the local and fully established laminar heat transfer of the aqueous CMC solutions in the square duct.

2. EXPERIMENTAL SYSTEM

Figure 2 shows the test section of the present experiments. The square ducts were cut in a base made of transparent material PMMA with a low conductivity of about 0.14 W m^{-1} K^{-1}. To ensure the accuracy of measurements, five uniformly placed square ducts of the same hydraulic diameter ($D_h = 0.4$ cm) were used. The middle one is taken as the object of interest. The top wall is a 0.9 mm thick stainless steel plate, and is heated from outside by a 0.08 mm thick electrothermal band under constant heat flux boundary conditions. The top cover was used to hold the rubber plate, top wall and the duct base together. The entrance to the square ducts is sharp, so the simultaneously developing flows can be generated. The ratio of the duct length to hydraulic diameter, L/D_d, is 600, thus fully developed flow and heat transfer results can be obtained.

The test section was adapted to the experimental set-up shown in Fig. 3. An a.c. unit with maximum a.c. power output of 5 kW was used as the power supply for the heat transfer experiments. The prepared test fluid in the bottom tank was transported to the top tank by a special G28-1 screw pump, which can minimize the mechanical rupture of the molecular structure of the polymer solutions. Driven by gravity force, the test fluid in the top tank then flowed into the lower test section, with constant liquid head being kept by a funnel. Before the test section was a calming section used to reduce the disturbance and obtain a steady laminar flow. The valve before the calming section can be adjusted to give the desired mass flow rate in the flow loop. The mixing section following the test section is used to provide an equalized bulk fluid temperature. After recovering the heat gain of the test fluid by a heat exchanger, the experimental set-up lets the fluid either go to the weighing tank for mass flow

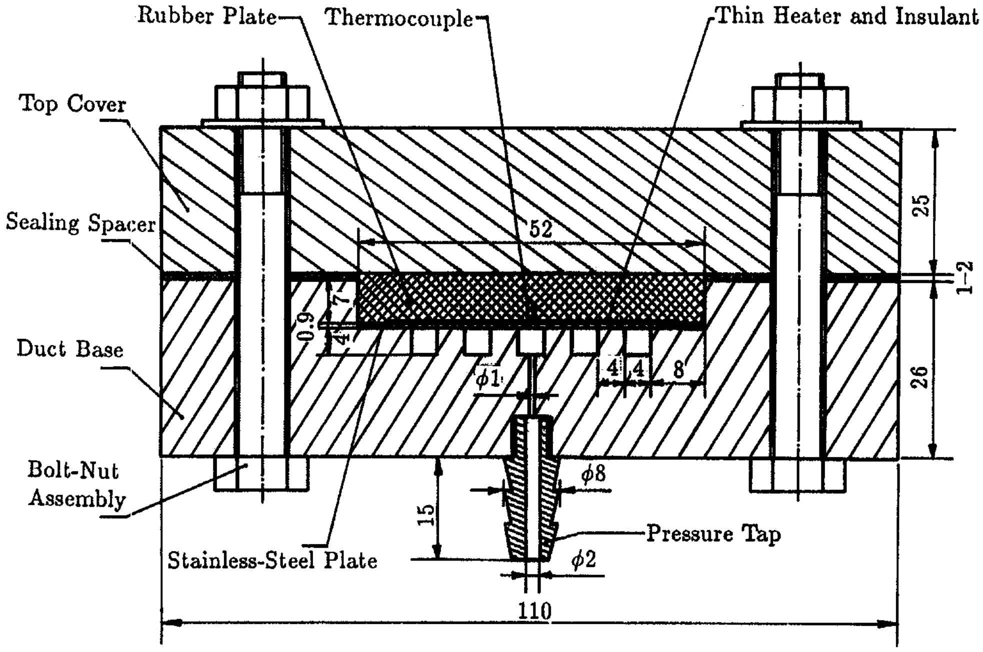

Fig. 2. Test section.

rate measurement or return to the bottom tank for recirculation.

The calming section, test section and mixing section were wrapped by 100 mm thick thermal insulation material with an effective conductivity of 0.019 W m^{-1} K^{-1}. It was estimated that the heat loss to the environment was considerably less than 1%. By measuring the electric current and voltage drop across the electrothermal band, the heat flux q per unit heating area was determined. The outside wall temperature distributions along the top wall of the square duct were measured by thirty-five 0.2 mm calibrated copper–constantan thermocouples located at different axial positions. Near the entrance in the calming section, the mean fluid temperature T_{bi} was measured. Close to the exit in the mixing section, the mean fluid temperature T_{bo} was measured. For the present thermal conditions, T_b, the bulk temperature at the different axial locations can be calculated from T_{bi} and T_{bo} by linear interpolation. The pressure taps located at 16 positions along the bottom wall of the duct were used for measuring the pressure drops with a water column manometer.

As only the top wall is heated, the buoyancy effects on the non-Newtonian fluid flow through the ducts should be very weak. An analysis has shown that the effects of the heat conduction from the duct walls are also very small within the investigated parameter ranges. Perhaps the most efficient way to evaluate the accuracy of the above experimental system is to perform experiments with standard Newtonian and non-Newtonian fluids on it and compare the experimental results with the published data about the laminar forced flow heat transfer available to the public. This has been done by Lin and Ko [18] and the experimental results with water and Carbopol solution are in excellent agreement with the standard available data, suggesting that the present experimental system can produce forced flow and heat transfer data with very good accuracy.

3. TEST FLUIDS

The non-Newtonian aqueous CMC solutions, which are viscoelastic, are prepared by adding to Beijing tap water the sodium carboxymethylcellulose (CMC). The CMC used, which is from Beijing Chemical Reagent Company, is a high-molecular polymer with linear structure and excellent solubility. Two aqueous CMC solutions with concentrations of 4000 and 8000 wppm were used in the present experimental study.

The viscometer used to measure the apparent viscosities of the non-Newtonian fluids in different shear rate ranges was a NXS-11 type rotational one in the Heat Transfer Laboratory, Institute of Engineering Thermophysics, Chinese Academy of Sciences. In order to clearly define the test fluids, the apparent viscosities of the fluids were measured before and after each heat transfer run.

Figure 4 gives a typical result of measured apparent viscosity η vs shear rate $\dot{\gamma}$ of the aqueous CMC solutions. Over the whole range of shear rate $\dot{\gamma}$ measured in the rotational viscometer ($\dot{\gamma} = 15 \sim 10^3$ s^{-1}), the apparent viscosity η of the 8000 wppm aqueous CMC solution is higher than that of the 4000 wppm aqueous CMC solution. The apparent viscosity η of the two

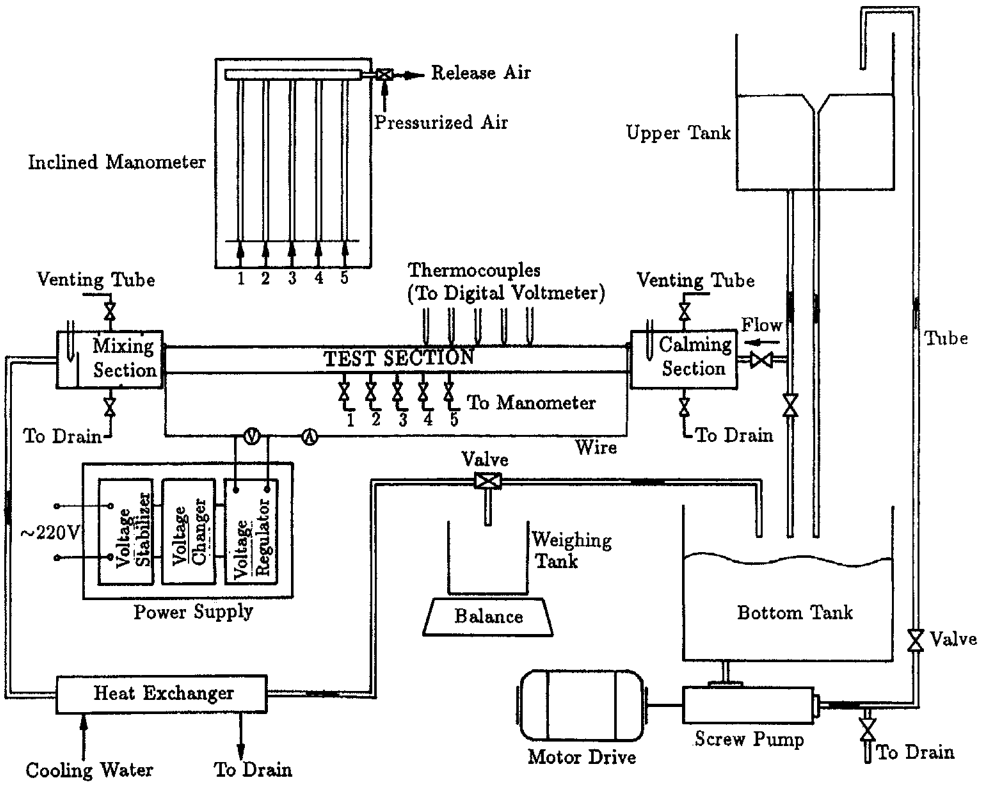

Fig. 3. Experimental set-up.

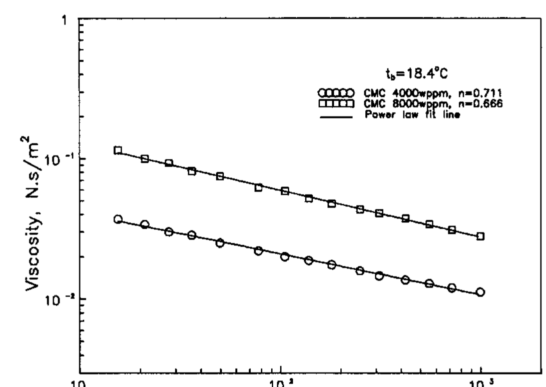

Fig. 4. Typical apparent viscosity vs shear rate of aqueous CMC solutions.

aqueous CMC solutions can be well fitted by the following power law within 4.5% :

$$\eta = K\dot{\gamma}^{n-1} \tag{1}$$

where K and n are consistency index and power-law index in the power-law model, respectively. For a fixed temperature condition, the values of K and n depend on the polymer concentration C.

In the present study, the range of the power-law index n for the investigated non-Newtonian fluids is 0.66–0.78. The consistency index K covers a range of 0.05–0.28 N s^n m^{-2}.

4. RESULTS AND DISCUSSION

In the present study, the pressure drop and the heat transfer rate of the aqueous CMC solutions in the square duct were measured simultaneously. The fluid temperature range for heat transfer study was kept within 16–23°C. The bulk fluid temperature was used to characterize the values of the fluid properties. To do a statistical correlation, the number of experimental runs is about 20 for each of the CMC solutions. The investigated range of Reynolds number (Re^*) is 1.0–500.

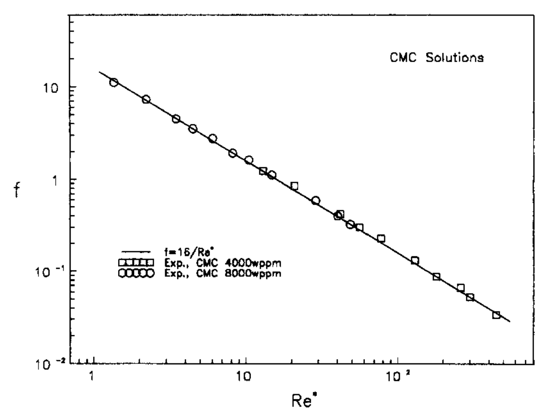

Fig. 5. Friction factor measurements for aqueous CMC solutions in fully developed laminar flow through a square duct.

4.1. *Fully developed friction factor*

The measured fully developed Fanning friction factor f for the aqueous CMC solutions as a function of Reynolds number Re^* is shown in Fig. 5. It is seen that the experimental data are in excellent agreement with the power-law prediction of Kozicki *et al.* [19] even at very low Reynolds numbers:

$$f = 16/Re^* \tag{2}$$

where Re^* is the Kozicki generalized Reynolds number, which for a square duct, can be written as

$$Re^* = \rho U^{2-n} D_h^n \Bigg/ \left[8^{n-1} K \left(0.6766 + \frac{0.2121}{n} \right)^n \right]. \tag{3}$$

It has been stated by Cohen and Metzner [20] and Lu *et al.* [21] that significant apparent slip may occur in some forced laminar flows of polymer solutions through capillary tubes of very small diameters. In the present study, the maximum deviation of the experimental friction factor from the power-law prediction [equation (2)] is 10%. No significant drag decrease was found in the hydrodynamically developing and developed regions. It seems that the 4000 and 8000 wppm aqueous CMC solutions did not exhibit significant apparent slip in the square duct of 0.4 cm hydraulic diameter.

4.2. *Local Nusselt number*

Figures 6 and 7 present the measured local Nusselt numbers Nu_x vs Graetz number Gz for the 4000 and 8000 wppm aqueous CMC solutions, respectively. Also shown is the Newtonian prediction of Wibulswas [22] for a square duct with all four walls heated under constant axial heat flux and constant local peripheral wall temperature, the $H1(4)$ boundary condition. Also, shown for reference is the fully developed Nusselt number (forced convection limit) [23] calculated for a Newtonian fluid in a square duct with only one of the duct walls heated, the $H1(1)$ boundary condition.

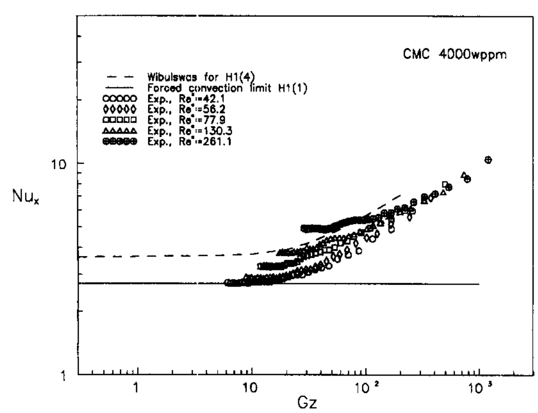

Fig. 6. Local Nusselt number vs Graetz number of 4000 wppm aqueous CMC solution as a function of Reynolds number.

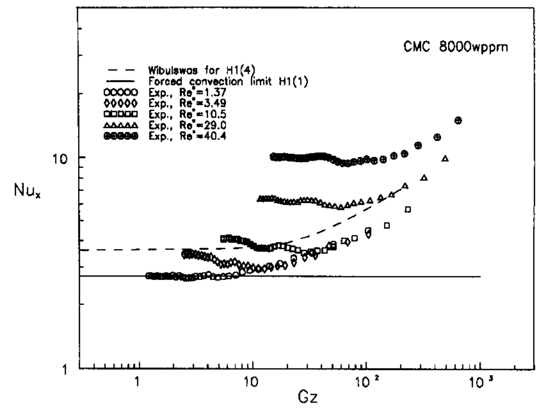

Fig. 7. Local Nusselt number vs Graetz number of 8000 wppm aqueous CMC solution as a function of Reynolds number.

It is very interesting that the aqueous CMC solutions, like the other non-Newtonian fluids used in the previous heat transfer studies, have the ability to augment the heat transfer in laminar flow through a non-circular duct. For a given polymer concentration, higher heat transfer increase with aqueous CMC solution can be produced at higher Reynolds number. Furthermore, to enhance the local heat transfer significantly, the Reynolds number needed must be high enough for the case of low polymer concentration (4000 wppm, see Fig. 6); but for the case of high polymer concentration (8000 wppm, see Fig. 7), the heat transfer can be dramatically increased even at very low Reynolds numbers.

In this paper, the authors intend to follow the opinion of Kostic [4] *et al.*, that it is the fluid elasticity giving rise to secondary flows that should be responsible for the observed large increase in the laminar heat transfer of aqueous CMC solutions in the investigated square duct. The results shown in Figs. 6 and 7 imply that the elastic effect of the 8000 wppm aqueous CMC solution on the laminar heat transfer in square duct is much stronger than that of the 4000 wppm aqueous CMC solution.

To give a quantitative analysis of the local heat transfer enhancement, the typical local Nusselt number results for the 4000 and 8000 wppm aqueous CMC solutions at a nearly constant Reynolds number (Re^* is about 41) are chosen as an example, and are shown in Fig. 8. Under the conditions marked in Fig. 8, the local Nusselt numbers of the two aqueous CMC solutions can be approximately fitted within 10% by the following exponential functions:

$$Nu_{x,4000\text{wppm}} = 2.714 \exp(0.00349 Gz) \tag{4}$$

$$Nu_{x,8000\text{wppm}} = 9.665 \exp(0.000601 Gz). \tag{5}$$

Then, a parameter E representing the relative variation of the local heat transfer between the two aqueous CMC solutions is defined as

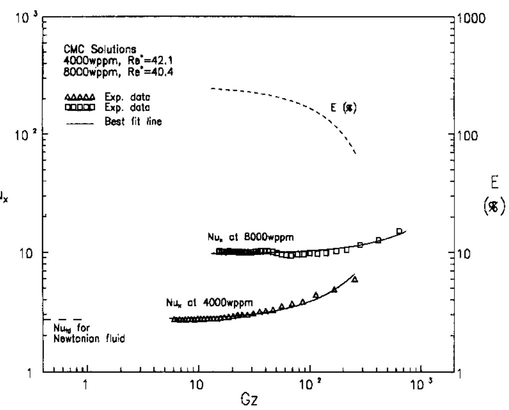

Fig. 8. Typical local Nusselt numbers with their relative change for two aqueous CMC solutions as a function of Graetz number at a nearly constant Reynolds number.

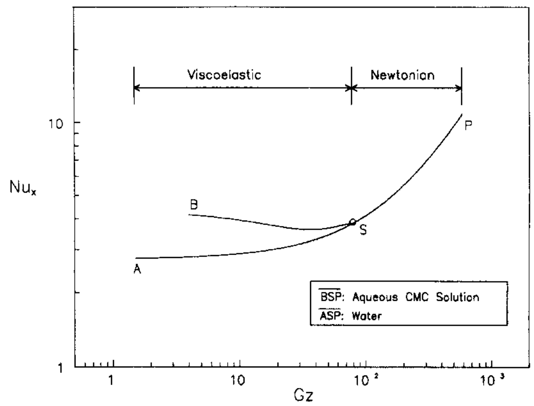

Fig. 9. Local heat transfer model for the non-Newtonian aqueous CMC solutions.

$$E = \frac{Nu_{x,8000\text{wppm}} - Nu_{x,4000\text{wppm}}}{Nu_{x,4000\text{wppm}}} \times 100\%$$
$$= [3.561\exp(-0.00289Gz) - 1.0] \times 100\%. \quad (6)$$

The above equation is most applicable within a Gz range of 15–255. As shown in Fig. 8, sharp increase of E takes place within the relatively higher Gz range. The values of E increase with decreasing Gz until fully developed heat transfer is achieved, where E remains constant.

Here, it should be noticed that the local Nusselt number for 4000 wppm aqueous CMC solution at $Re^* = 42.1$ (given in Fig. 8) is almost equal to that for water, with the values of the former just less than 1.5% higher than those of the latter under the same conditions [18]. So, the parameter E defined in equation (6) can also be considered as the relative heat transfer increase of the 8000 wppm aqueous CMC solution over Newtonian fluid, water. One of the typical results is that when the value of Gz decreases from 250 to 15, the value of the relative heat transfer enhancement of the 8000 wppm aqueous CMC solution, i.e. the value of E, increases from 72.9 to 241.0%. It seems that the maximum heat transfer enhancement occurs at the fully developed heat transfer region.

Although the local Nusselt number Nu_x is a very complex function of the polymer concentration C, Reynolds number Re^* and Graetz number Gz, careful examination of the distributions of Nu_x has made it possible for the authors to propose a general model to be helpful in the understanding of the complex local heat transfer of the aqueous CMC solutions in the non-circular duct. The proposed local heat transfer model for the non-Newtonian fluid in a square duct is given in Fig. 9, where the local Nusselt number for the aqueous CMC solution $\overline{PSB}$ over the whole Gz range is divided into two sections, which are named "Newtonian heat transfer section" and "viscoelastic heat transfer section", respectively. Within the Newtonian section, the local Nusselt number of aqueous CMC solution $\overline{PS}$ is almost the same as that of Newtonian fluid (water). But within the viscoelastic section, the local Nusselt number of aqueous CMC solution $\overline{SB}$ is higher than that of Newtonian fluid $\overline{SA}$.

In the above two-zone model for local heat transfer, the point "S" is a very important item. It is hereby termed "critical point", in view of the fact that the non-Newtonian fluid flow is able to generate secondary flow strong enough to give considerable heat transfer increase over Newtonian fluid when the following condition is satisfied:

$$Gz \leqslant Gz_S \quad (7)$$

where Gz_S is the Graetz number corresponding to the critical point "S". When the polymer concentration or Reynolds number are increased, the critical point S will move toward the point P along the line $\overline{ASP}$, which is the local Nusselt number for Newtonian fluid.

As the local heat transfer of Newtonian fluid is very easy to determine, the complexity of the local heat transfer of the non-Newtonian fluid is now concentrated on the description of the local Nusselt number $\overline{SB}$ within the viscoelastic section. Generally speaking, the pattern of the local Nusselt number $\overline{SB}$ will depend on fluid elasticity, flow condition, and geometric location. In the present study, it was found that when the Reynolds number is relatively high enough for a given concentration of aqueous CMC solution, the local Nusselt number $\overline{SB}$ within the viscoelastic section will remain nearly constant and depends only on the value of Reynolds number Re^*.

4.3. *Fully developed Nusselt number*

Figure 10 presents the fully developed Nusselt number Nu_{fd} vs Reynolds number Re^* for the 4000 and 8000 wppm aqueous CMC solutions. For each of the CMC solution, it is found that the Nusselt number Nu_{fd} can be well described by the following linear correlation:

$$Nu_{fd} = a + b(Re^*) \quad (8)$$

where the constants a and b are 2.180 and 0.0119,

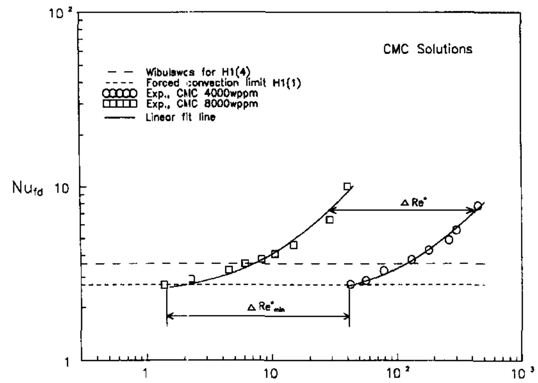

Fig. 10. Fully developed Nusselt numbers for aqueous CMC solutions as a function of Reynolds number.

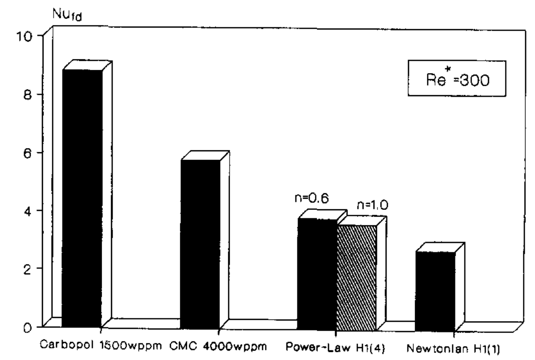

Fig. 11. Typical comparison of the fully developed Nusselt number of an aqueous CMC solution with an aqueous Carbopol solution.

respectively for the 4000 wppm CMC solution, and 2.353 and 0.173, respectively for the 8000 wppm CMC solution. The maximum deviation of the experimental Nusselt number from the prediction by equation (8) is 12%. Within the investigated Re^* range, equation (8) is valid when the following requirement is satisfied:

$$Re^* \geqslant Re^*_{min} \tag{9}$$

where Re^*_{min} is defined as the minimum Kozicki Reynolds number required by the non-Newtonian fluid to start generating higher heat transfer than Newtonian fluid. Careful experiments have revealed that the value of Re^*_{min} is about 40 for 4000 wppm CMC solution, and 1.4 for 8000 wppm CMC solution.

Consequently, ΔRe^*, the difference between the Reynolds number needed by 4000 wppm aqueous CMC solution and that needed by 8000 wppm aqueous CMC solution to produce the same level of fully developed heat transfer, can also be determined by a linear function:

$$\Delta Re^* = 77.45 Nu_{fd} - 167.85. \tag{10}$$

As the minimum value of Nu_{fd} for aqueous CMC solutions is equal to the value of the fully developed Nusselt number for Newtonian fluid, the lowest value of ΔRe^* in equation (10) is the same as ΔRe^*_{min}, the difference between the minimum Re^* required for 4000 wppm aqueous CMC solution and that for 8000 wppm CMC solution to start enhancing heat transfer in the laminar duct flow.

4.4. *Further discussion*

As the polymer CMC is a new kind of additive applied in the experimental study of laminar heat transfer in a non-circular duct, it is useful to give a brief estimation of the aqueous CMC solution's capability relative to other additives to enhance heat transfer in laminar duct flow. Figure 11 makes a comparison of the fully developed Nusselt number Nu_{fd} for 4000 wppm aqueous CMC solution in the duct to that for 1500 wppm neutralized aqueous Carbopol solution [18] under the same experimental conditions. Also shown in Fig. 11 are the computational result for power-law fluid under $H1(4)$ boundary condition and that for Newtonian fluid under $H1(1)$ boundary condition.

It is shown from Fig. 11 that both of the viscoelastic aqueous polymer solutions have substantially higher ability to enhance heat transfer than the purely viscous power-law fluid and Newtonian fluid. At the given Reynolds number ($Re^* = 300$), the Nusselt number Nu_{fd} of the aqueous CMC solution at a concentration of 4000 wppm is less than that of the aqueous Carbopol solution at an even lower concentration of 1500 wppm, indicating that for the case of fixed polymer concentration, the elastic effect of the aqueous CMC solution on the laminar heat transfer enhancement in non-circular duct flow would, in general, be weaker than that of the aqueous Carbopol solution.

5. CONCLUSION

Two aqueous carboxymethylcellulose (CMC) solutions have been employed to investigate the laminar flow heat transfer in a top-heated square duct of a relatively small hydraulic diameter. It has been found that although their elastic effect is not stronger than that of the aqueous Carbopol solution, the aqueous CMC solutions can also produce significant heat transfer enhancement at certain values of polymer concentration and Reynolds number.

A local heat transfer model is proposed based on the present experimental data. The main feature of the model is that the local heat transfer of the non-Newtonian fluids along the whole duct wall can be described by a two-zone structure which is composed of a Newtonian section and a viscoelastic one. The laminar flows of the aqueous CMC solutions can possess the ability to create considerable local heat transfer enhancement within the viscoelastic section where the Graetz numbers are lower than a critical value.

For the given physical conditions with aqueous CMC solutions, the thermally developed laminar flows generate the maximum enhancement of heat transfer. The fully established heat transfer correlation for each of the aqueous CMC solutions was found to be a linear function of the Kozicki generalized Reynolds number Re^* within the investigated parameter ranges.

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